Section 2.3

The Division Algorithm

If $f(x)$ and $d(x)$ are polynomials such that $d(x) \neq 0$, and the degree of $d(x)$ is less than or equal to the degree of $f(x)$, then there exist unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = d(x)q(x) + r(x), \quad \text{or} \quad \frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)},$$

where $r(x) = 0$ or the degree of $r(x)$ is less than the degree of $d(x)$. If the remainder $r(x)$ is zero, $d(x)$ *divides evenly* into $f(x)$.

The Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, the remainder is $r = f(k)$.

The Factor Theorem

A polynomial $f(x)$ has a factor $(x - k)$ if and only if $f(k) = 0$.

The Remainder in Synthetic Division

The remainder r, obtained in the synthetic division of $f(x)$ by $x - k$, provides the following information.

a) The remainder r gives the value of f at $x = k$. That is, $r = f(k)$.

b) If $r = 0$, $(x - k)$ is a factor of $f(x)$.

c) If $r = 0$, $(k, 0)$ is an x-intercept of the graph of f.

Problem 1. Use long division to divide.

a) $(6x^3 - 16x^2 + 17x - 6) \div (3x - 2)$

b) $(x^4 + 3x^2 + 1) \div (x^2 - 2x + 3)$

Problem 2. Use synthetic division to divide.

a) $(5x^3 + 18x^2 + 7x - 6) \div (x + 3)$

b) $(3x^3 - 16x^2 - 72) \div (x - 6)$

Problem 3. Write the given function in the form $f(x) = (x - k)q(x) + r$ for the given value of k, and demonstrate that $f(k) = r$.

a) $f(x) = x^3 - 5x^2 - 11x + 8,\ k = -2$

b) $f(x) = 10x^3 - 22x^2 - 3x + 4,\ k = \frac{1}{5}$

Problem 4. Use synthetic division to show that $x = -4$ is a solution of $x^3 - 28x - 48 = 0$, and use the result to factor the polynomial completely.

Problem 5. Verify the given factors of the function f, find the remaining factors of f, use your results to write the complete factorization of f, and list all zeros of f.

a) $f(x) = 3x^3 + 2x^2 - 19x + 6$, factors $(x+3), (x-2)$

b) $f(x) = 8x^4 - 14x^3 - 71x^2 - 10x + 24$, factors $(x+2), (x-4)$

Homework: Read section 2.3, do #11, 15, 23, 27, 31, 35, 47, 55, 59, 69, 81 (the quiz for this section will be taken from these problems)